Minimal Spanning Trees

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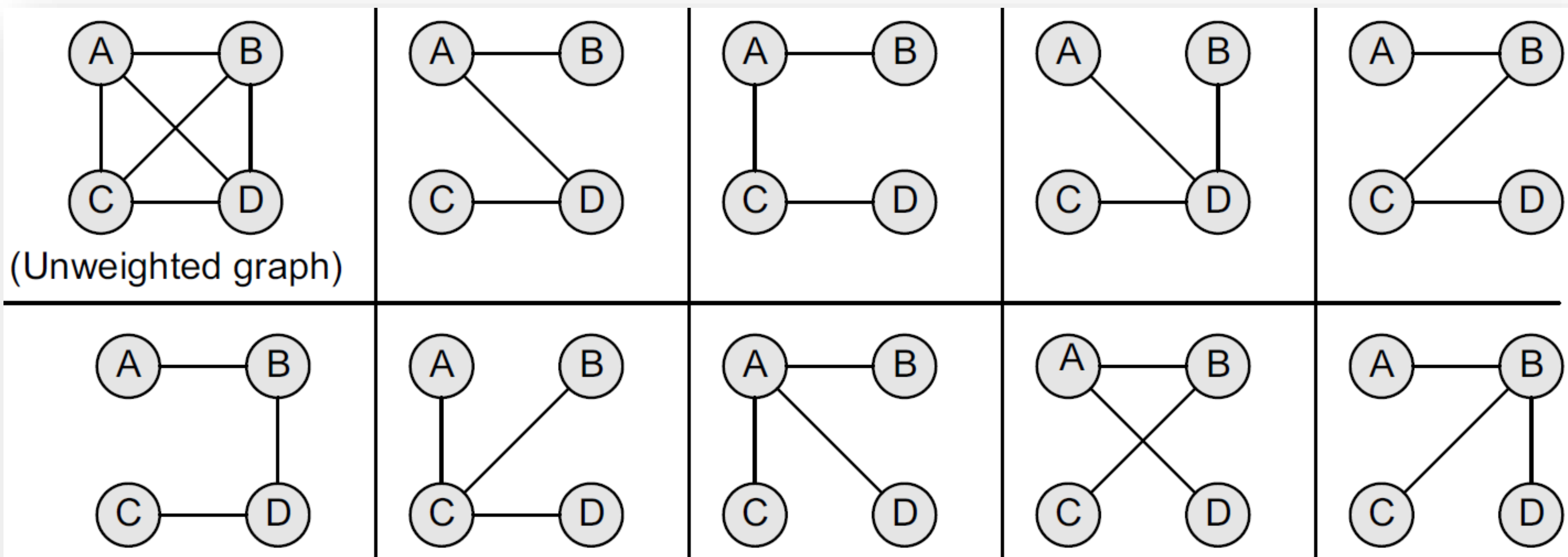
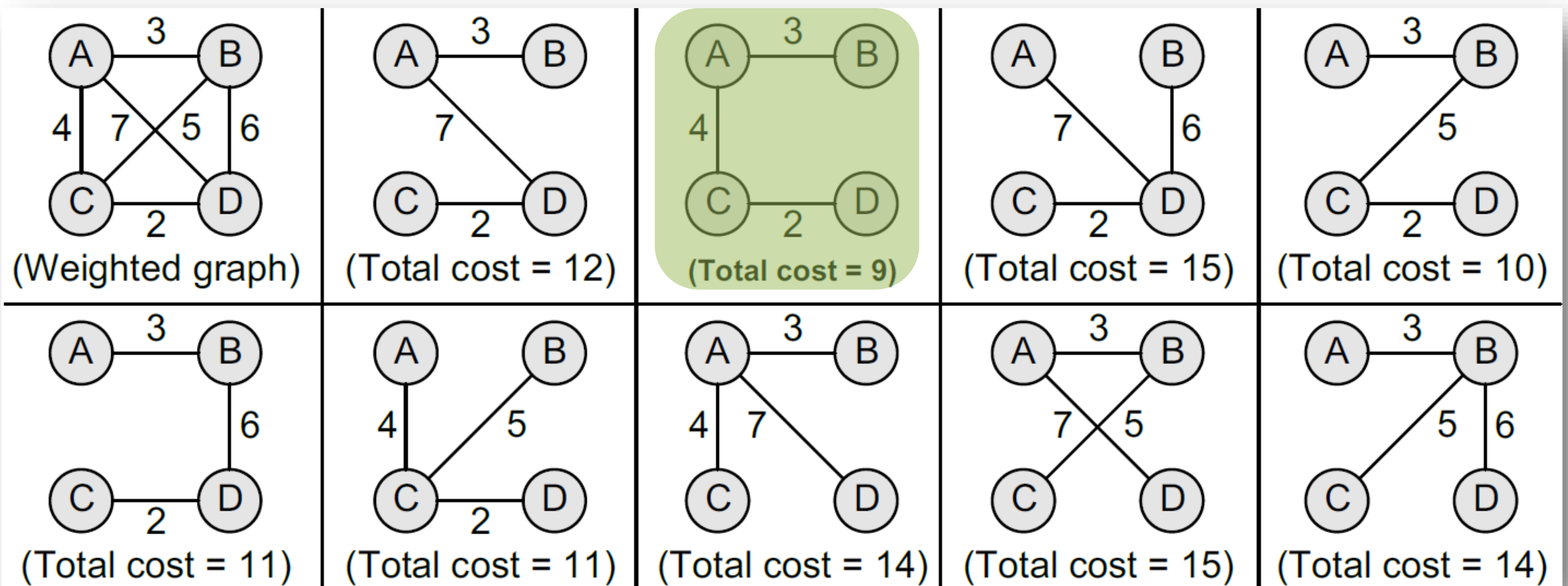
Spanning Tree

- A spanning tree of a connected, undirected graph G is a sub-graph of G which is a tree that connects all the vertices together
  - A graph G can have many different spanning trees



Minimum Spanning Tree

- A **minimum spanning tree** (MST) is defined as a spanning tree with weight less than or equal to the weight of every other spanning tree
  - We can assign **weights** to each edge, and use it to assign a weight to a spanning tree by calculating the sum of the weights of the edges in that spanning



Prim's Algorithm.

- Prim's algorithm is a greedy algorithm that is used to form a minimum spanning tree for a connected weighted undirected graph
  - **Tree vertices**
    - Vertices that are a part of the minimum spanning tree *T*
  - **Fringe (Neighboring) vertices**
    - Vertices that are currently not a part of *T*, but are adjacent to some tree vertex
  - **Unseen vertices**
    - Vertices that are neither tree vertices nor fringe vertices fall under this category

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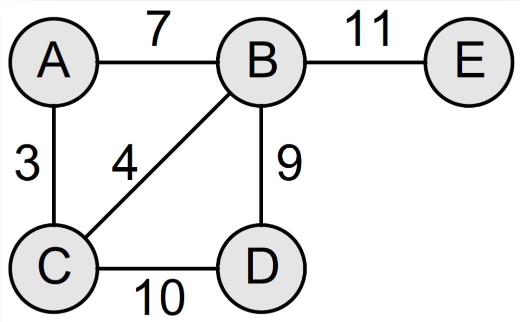
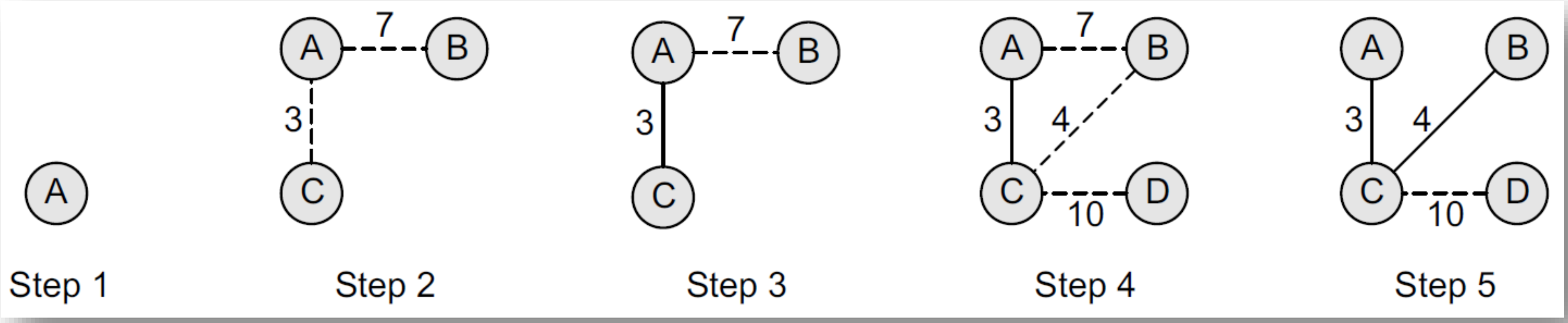
Step 1: Select a starting vertex
Step 2: Repeat Steps 3 and 4 until there are fringe vertices
Step 3:     Select an edge e connecting the tree vertex and
            fringe vertex that has minimum weight
Step 4:     Add the selected edge and the vertex to the
            minimum spanning tree T
        [END OF LOOP]
Step 5: EXIT
```

Prim's Algorithm..

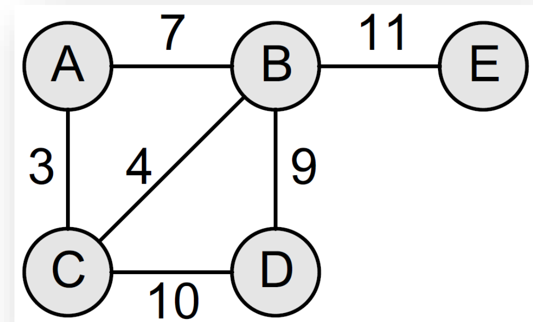
- Construct a minimum spanning tree of the graph by using Prim's algorithm
  - Step 1: Choose a starting vertex *A*
  - Step 2: Add the fringe vertices (that are adjacent to *A*)
  - Step 3: Since the edge connecting *A* and *C* has less weight, add *C* to the tree
  - Step 4: Add the fringe vertices (that are adjacent to *C*)
  - Step 5: Since the edge connecting *C* and *B* has less weight, add *B* to the tree

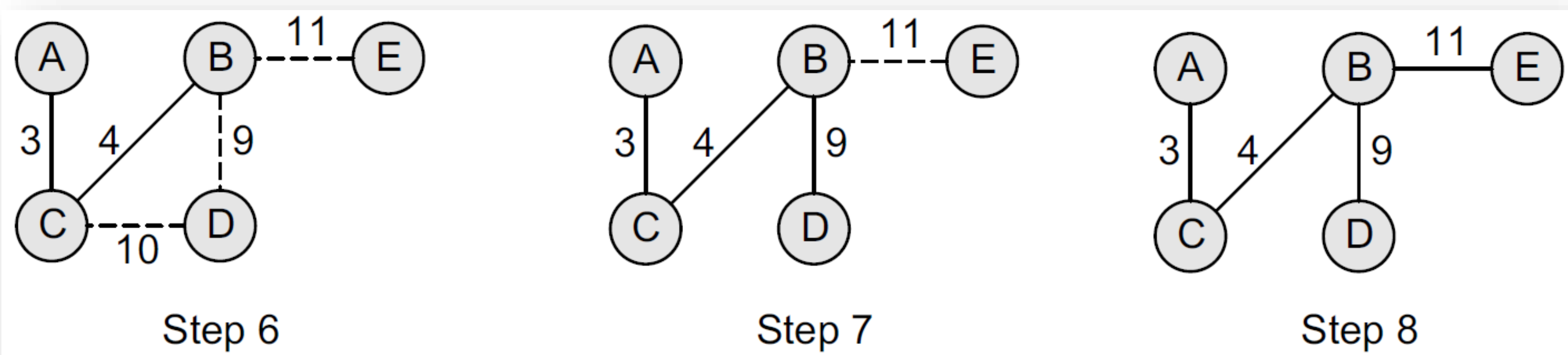




Prim's Algorithm…

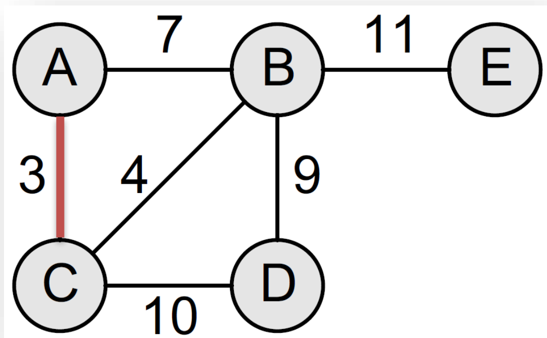
- Step 6: Add the fringe vertices (that are adjacent to B)
- Step 7: Since the edge connecting B and D has less weight, add D to the tree
- Step 8: Add E to the tree

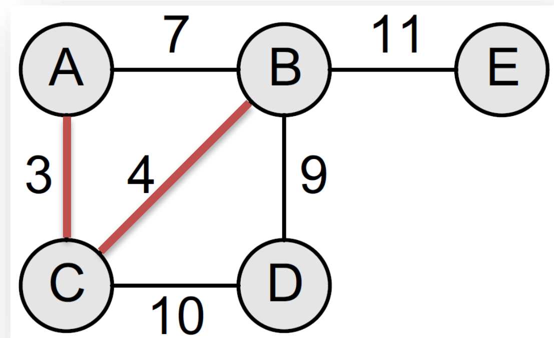


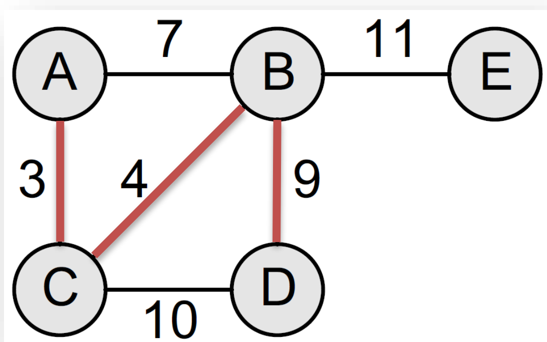


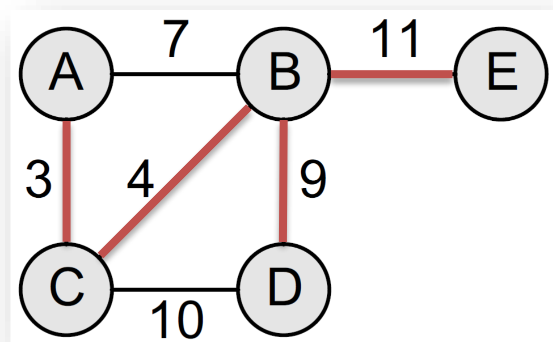
Prim’s Algorithm….

- By looking!



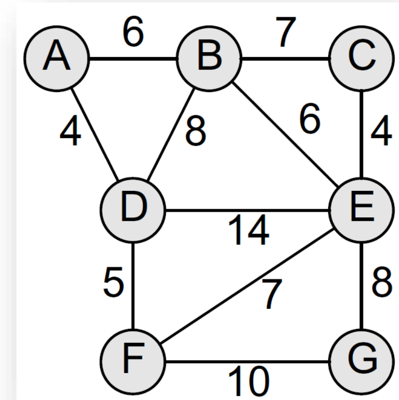


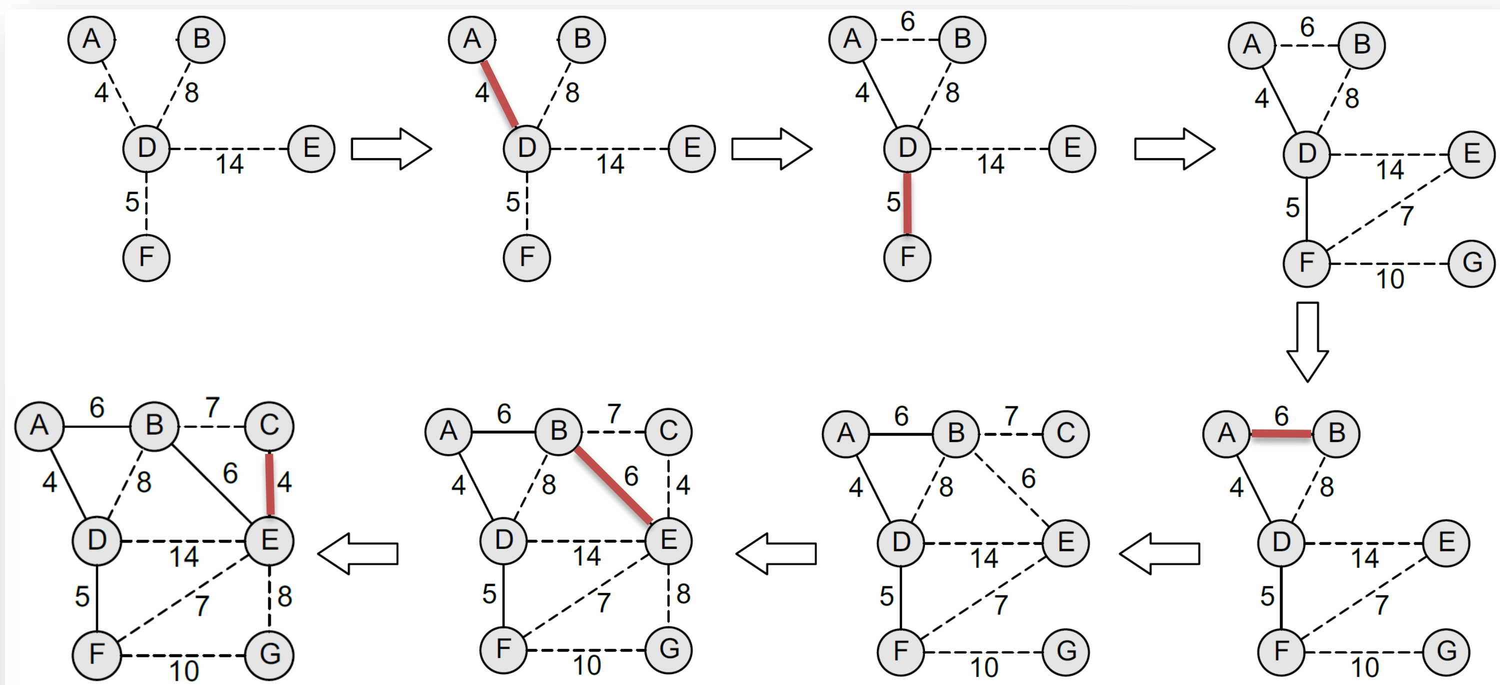


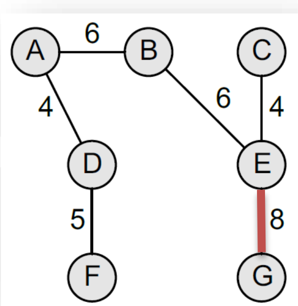


Prim’s Algorithm…..

- Construct a minimum spanning tree of the graph by using Prim’s algorithm from vertex *D*







Kruskal's Algorithm.

- Kruskal's algorithm is used to find the minimum spanning tree for a connected weighted graph
  - If the graph is not connected, then it finds a **minimum spanning forest**

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Step 1: Create a forest in such a way that each graph is a separate
        tree.
Step 2: Create a priority queue Q that contains all the edges of the
        graph.
Step 3: Repeat Steps 4 and 5 while Q is NOT EMPTY
Step 4:      Remove an edge from Q
Step 5: IF the edge obtained in Step 4 connects two different trees,
        then Add it to the forest (for combining two trees into one
        tree).
        ELSE
             Discard the edge
Step 6: END
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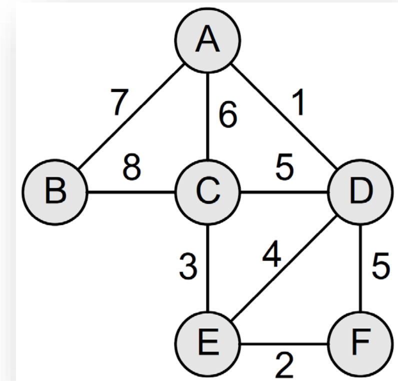
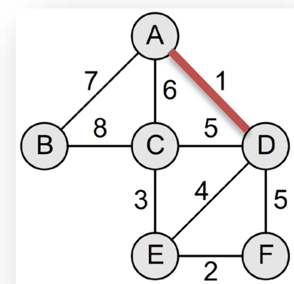
Kruskal's Algorithm..

- Apply Kruskal's algorithm on the given graph



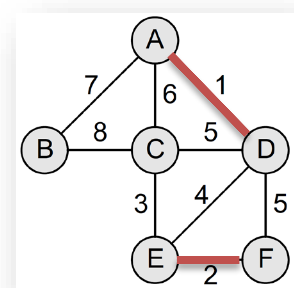
  - Initial:
    - F = {{A}, {B}, {C}, {D}, {E}, {F}}
    - MST = {}
    - Priority Queue Q = {(A, D), (E, F), (C, E), (E, D) (C, D), (D, F), (A, C), (A, B), (B, C)}
  - Step1:
    - Remove the edge (A, D) from Q

F = {{A, D}, {B}, {C}, {E}, {F}}
MST = {A, D}
Q = {(E, F), (C, E), (E, D), (C, D), (D, F), (A, C), (A, B), (B, C)}



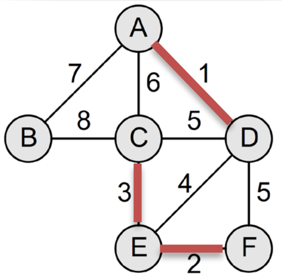
  - Step2:
    - Remove the edge (E, F) from Q

F = {{A, D}, {B}, {C}, {E, F}}
MST = {(A, D), (E, F)}
Q = {(C, E), (E, D), (C, D), (D, F), (A, C), (A, B), (B, C)}



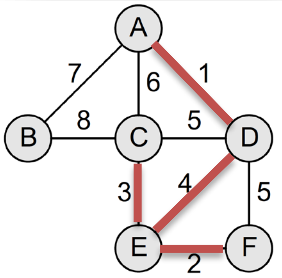
Kruskal's Algorithm...

- Step3:
  - Remove the edge (C, E) from Q



F = {{A, D}, {B}, {C, E, F}}
MST = {(A, D), (C, E), (E, F)}
Q = {(E, D), (C, D), (D, F), (A, C), (A, B), (B, C)}

- Step4:
  - Remove the edge (E, D) from Q



F = {{A, C, D, E, F}, {B}}
MST = {(A, D), (C, E), (E, F), (E, D)}
Q = {(C, D), (D, F), (A, C), (A, B), (B, C)}

- Step5:
  - Remove the edge (C, D) from Q

    The edge does not connect different trees, so simply discard this edge

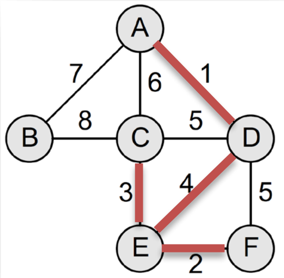
F = {{A, C, D, E, F}, {B}}
MST = {(A, D), (C, E), (E, F), (E, D)}
Q = {(D, F), (A, C), (A, B), (B, C)}

Kruskal's Algorithm….

– Step6:
  - Remove the edge (D, F) from Q

    The edge does not connect different trees, so simply discard this edge

$$F = \{\{A, C, D, E, F\}, \{B\}\}$$
$$MST = \{(A, D), (C, E), (E, F), (E, D)\}$$
$$Q = \{(A, C), (A, B), (B, C)\}$$



– Step7:
  - Remove the edge (A, C) from Q

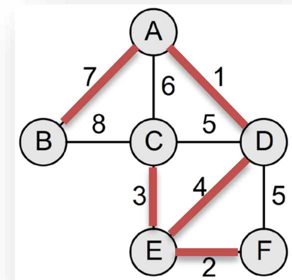
    The edge does not connect different trees, so simply discard this edge

$$F = \{\{A, C, D, E, F\}, \{B\}\}$$
$$MST = \{(A, D), (C, E), (E, F), (E, D)\}$$
$$Q = \{(A, B), (B, C)\}$$

Kruskal’s Algorithm…..

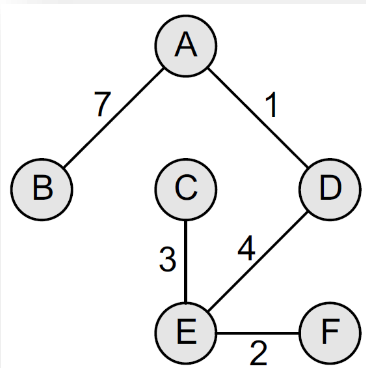
- Step8:
  - Remove the edge (A, B) from Q

F = {A, B, C, D, E, F}
MST = {(A, D), (C, E), (E, F), (E, D), (A, B)}
Q = {(B, C)}



- Step8:
  - Remove the edge (B, C) from Q

  The edge does not connect different trees, so simply discard this edge



F = {A, B, C, D, E, F}
MST = {(A, D), (C, E), (E, F), (E, D), (A, B)}
Q = {}

Questions?

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